REFLECTION OF MAGNETOACOUSTIC WAVES FROM AN ELASTIC LAYER WITH FINITE ELECTRIC CONDUCTIVITY

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The problem of magnetoacoustic wave reflection from a plane layer of electrically conducting liquid (or gas) in a constant uniform magnetic field R was solved in [1].

In the following we solve the analogous problem for an elastic layer. The layer reflection and transmission coefficients are found in the limiting cases of weak and strong magnetic fields.

1. We examine reflection from an elastic plane layer of thickness d on which a fast magnetoacoustic wave impinges at an arbitrary angle (figure). The edge of the layer coincides with the xy plane. The wave incidence plane is aligned with the xz axis. We assume that the vector H lies in this same plane and forms the angle φ with the x axis The liquid (or gaseous) media on both sides of the layer are electrically conductive.

In the liquid medium there are fast and slow magnetoacoustic waves, polarized in the xz plane, and Alfven waves, polarized perpendicular to this plane.

In the elastic medium there are five types of waves. Three of these – the fast and slow magnetoelastic and also the electromagnetic, associated with the process of magnetic field diffusion in the medium – are polarized in the xz plane. The fourth and fifth waves are polarized perpendicular to the xz plane.

The waves which are polarized perpendicular to the plane of incidence propagate independently of the others, therefore we shall not consider them in this study.

If a fast magnetoacoustic wave strikes the layer at the arbitrary angle θ, then both fast and slow magnetoacoustic waves will be reflected into the upper medium and will pass into the lower medium. Resultant waves of three types are formed within the layer as a result of multiple reflections from its boundaries.

We shall denote the medium from which the wave is incident, the layer, and the lower medium by the numerals 3, 2, and 1, respectively. We shall denote the quantities relating to the different wave types by letters with two subscripts $A_{\mu\nu}$, where $\mu = 1, 2, 3$ denotes the number of the medium in which the wave propagates, $\nu = 1$ corresponds to the first wave type, $\nu = 2$ is the second type, and $\nu = 3$ is the third type. We shall use a prime to denote quantities relating to waves propagating in the positive z-axis direction.

From the MHD equations for plane waves in a fluid follow the relations

$$\begin{gathered}
v_{\mu\nu x} = M_{\nu\mu} v_{\mu\nu z}, \quad h_{\mu\nu x} = A_{\mu\nu} v_{\mu\nu z}, \quad E_{\mu\nu y} = B_{\mu\nu} v_{\mu\nu z}, \quad p_{\mu\nu} = Z_{\mu\nu} v_{\mu\nu z} \\
M_{\mu\nu} = (k_{\mu\nu z} \cos \alpha_{\mu\nu} - k_{\mu\nu} u_{\mu\nu} \sin \varphi) / \beta_{\mu\nu}, \quad \beta_{\mu\nu} = k_{\mu\nu} u_{\mu\nu} \cos \varphi - k_{\mu\nu x} \cos \alpha_{\mu\nu} \\
A_{\mu\nu} = H (u_{\mu\nu} - 1) \sqrt{u_{\mu\nu}}\, k_{\mu\nu z} / a_\mu \psi_\mu \beta_{\mu\nu}, \quad B_{\mu\nu} = -\omega A_{\mu\nu} / c\, k_{\mu\nu z} \\
Z_{\mu\nu} = -\rho_\mu a_\mu (k_{\mu\nu x} M_{\mu\nu} + k_{\mu\nu z}) / \omega, \quad \alpha_{\mu\nu} = 90^\circ - (\theta_{\mu\nu} - \varphi), \qquad \alpha_{\mu\nu}' = 90^\circ - (\theta_{\mu\nu}' + \varphi) \\
\psi_\mu = H^2 / 4\pi\rho_\mu a_\mu^2, \qquad u_{\mu\nu} = (\omega / k_{\mu\nu} a_\mu)^2, \quad (\mu = 1, 3; \nu = 1, 2.)
\end{gathered} \tag{1.1}$$

Here v_z, p, and ρ are the velocity component, hydrodynamic pressure, and density of the fluid, h is a small change of the magnetic field intensity in the wave, E_y is the intensity of the induced electric field,

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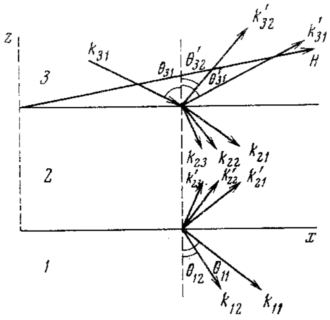

Fig. 1

k is the wave vector, ω is frequency, a is the sound speed in the fluid, c is the speed of light, α is the angle between the vectors k and H, and u and ψ are the squares of the phase velocity and the magnetic field intensity in dimensionless form.

The phase velocities of the magnetoacoustic waves are defined by the dispersion equation

$$u_\mu^2 - (1+\psi_\mu)\,u_\mu + \psi_\mu \cos^2\alpha_\mu + i\omega\eta_\mu (u_\mu - 1) = 0\,, \quad \eta_\mu = c^2/4\pi\sigma_\mu a_\mu^2 \tag{1.2}$$

where σ_μ is the elecric conductivity of the medium.

In the case of highly conductive media $\omega\eta_\mu \ll 1$ and weak magnetic field $\psi_\mu \ll 1$ we find from (1.2)

$$u_{\mu 1} = 1 + \psi_\mu \sin^2\alpha_{\mu 1}, \qquad u_{\mu 2} = \psi_\mu \cos^2\alpha_{\mu 2} - i\omega\eta_\mu \tag{1.3}$$

For a strong magnetic field $\psi_\mu \gg 1$ we have

$$u_{\mu 1} = \psi_\mu + \sin^2\alpha_{\mu 1}, \qquad u_{\mu 2} = \cos^2\alpha_{\mu 2}\left(1 - \frac{1}{\psi_\mu}\sin^2\alpha_{\mu 2}\right) \tag{1.4}$$

For waves in an elastic medium we have the relations [2]

$$\begin{gathered} v_{2\nu x} = M_{2\nu} v_{2\nu z}\;\; h_{2\nu x} = A_{2\nu} v_{2z\nu}\;\; E_{2\nu y} = B_{2\nu} v_{2\nu z},\;\; P_{2\nu zz} = Z_{2\nu} v_{2\nu z}, \qquad P_{2\nu xz} = X_{2\nu} v_{2\nu z} \\ M_{2\nu} = [(1-\xi)\,k_{2\nu z}\cos\alpha_{2\nu} - k_{2\nu}(u_{2\nu}-\xi)\sin\varphi]/\beta_{2\nu}\,, \quad \beta_{2\nu} = k_{2\nu}(u_{2\nu}-\xi)\cos\varphi - (1-\xi)\,k_{2\nu x}\cos\alpha_{2\nu} \\ A_{2\nu} = H(u_{2\nu}-1)(u_{2\nu}-\xi)\,k_{2\nu z}/a_2\psi_2\beta_{2\nu}\sqrt{u_{2\nu}}\,, \quad B_{2\nu} = -\omega A_{2\nu}/ck_{2\nu z}\,, \quad Z_{2\nu} = -\rho_2 a_2 [k_{2\nu z} + (1-2\xi)\,k_{2\nu x} M_{2\nu}]/\omega \\ X_{2\nu} = -\rho_2 b^2 (k_{2\nu x} + k_{2\nu z} M_{2\nu}), \qquad a_2^2 = (\lambda + 2\mu)/\rho_2\,, \quad b^2 = \mu/\rho_2, \qquad \xi = b^2/a_2^2, \qquad \psi_2 = H^2/4\pi\rho_2 a_2^2 \end{gathered} \tag{1.5}$$

The dispersion equation has the form [3, 4]

$$\begin{gathered} u_2^2 - (1+\xi+\psi_2)\,u_2 + \xi + \psi_2(\cos^2\alpha_2 + \xi\sin^2\alpha_2) + i\omega\,\eta_2 u_2^{-1}(u_2 - 1)(u_2 - \xi) = 0 \\ u_2 = (\omega/k_2 a_2)^2, \qquad \eta_2 = c^2/4\pi\sigma_2 a_2^2 \end{gathered} \tag{1.6}$$

For small $\omega\eta_2$ and ψ_2 the roots of this equation will be

$$u_{21} = 1 + \psi_2 \sin^2\alpha_{21}, \qquad u_{22} = \xi + \psi_2\cos^2\alpha_{22}, \qquad u_{23} = -i\omega\eta_2 \tag{1.7}$$

In the case $\psi_2 \gg 1$ we have

$$u_{21} = \psi_2 + \sin^2\alpha_{21} + \xi\cos^2\alpha_{21}, \qquad u_{22} = \cos^2\alpha_{22} + \xi\sin^2\alpha_{22} - \frac{(1-\xi)^2}{4\psi_2}\sin^2 2\alpha_{22}, \qquad u_{23} = -\frac{i\xi\omega\eta_2}{\psi_2(\cos^2\alpha_{23} + \xi\sin^2\alpha_{23})} \tag{1.8}$$

The angles $\theta_{\mu\nu}$ are connected by the relations (Snell's law)

$$k_{\mu\nu}\sin\theta_{\mu\nu} = k_{31}\sin\theta_{31} \tag{1.9}$$

According to (1.3) and (1.7), in the case of a weak magnetic field (1.9) takes the form

$$\frac{\sin\theta_{31}}{a_3} = \frac{\sin\theta_{\mu 1}}{a_\mu} = \frac{\sin\theta_{12}}{a_1\sqrt{\psi_1\sin^2\varphi - i\omega\eta_1}} = \frac{\sin\theta_{22}}{b} = \frac{\sin\theta_{23}}{a_2\sqrt{-i\omega\eta_2}} = \frac{\sin\theta_{32}}{a_3\sqrt{\psi_3\sin^2\varphi - i\omega\eta_3}}\,, \quad \theta_{\mu\nu}' = \theta_{\mu\nu} \tag{1.10}$$

For a strong field we have with account for (1.4) and (1.8)

$$\frac{\sin\theta_{31}}{a_3\sqrt{\psi_3}} = \frac{\sin\theta_{\mu 1}}{a_\mu\sqrt{\psi_\mu}} = \frac{\sin\theta_{12}}{a_1\sin\varphi} = \frac{\sin\theta_{22}}{a_2\sqrt{\sin^2\varphi + \xi\cos^2\varphi}} = \frac{\sin\theta_{23}\sqrt{\psi_2(\sin^2\varphi + \xi\cos^2\varphi)}}{a_2\sqrt{-i\xi\omega\eta_2}} = \frac{\sin\theta_{32}}{a_3\sin\varphi}, \quad \theta_{\mu\nu}' = \theta_{\mu\nu} \tag{1.11}$$

Taking the amplitude v_{31z} as unity, we write the velocity field in the form

$$\begin{gathered} v_{3z} = -\exp[-i\gamma_{31}(z-d)] + \sum_{\nu=1}^{2} W_{3\nu}'\exp[i\gamma_{3\nu}(z-d)]\,, \quad v_{2z} = \sum_{\nu=1}^{3}[W_{2\nu}\exp(-i\gamma_{2\nu}z) + W_{2\nu}'\exp(i\gamma_{2\nu}z)] \\ v_{1z} = -\sum_{\nu=1}^{2} W_{1\nu}\exp(-i\gamma_{1\nu}z), \qquad \gamma_{\mu\nu} = k_{\mu\nu}\cos\theta_{\mu\nu} \end{gathered} \tag{1.12}$$

For brevity we have dropped the common factor $\exp[ik_{\mu\nu}x - \omega t]$; $W_{3\nu}$ and $W_{1\nu}$ are the layer reflection and transmission amplitude coefficients, subject to determination.

The expressions for $h_{\mu x}$, $E_{\mu y}$, p_μ, P_{2zz}, P_{2xz} are obtained in accordance with (1.1), (1.5) by replacing the coefficients $W_{\mu\nu}$ in (1.12) by

$$A_{\mu\nu}W_{\mu\nu},\ B_{\mu\nu}W_{\mu\nu},\ Z_{\mu\nu}W_{\mu\nu},\ X_{\mu\nu}W_{\mu\nu}.$$

At the edges of the layer we have the conditions:

for z = 0

$$v_{1z} = v_{2z},\quad h_{1x} = h_{2x},\quad E_{1y} = E_{2y},\quad -p_1 = P_{2zz},\quad P_{2xz} = 0$$

for z = d (1.13)

$$v_{2z} = v_{3z},\quad h_{2x} = h_{3x},\quad E_{2y} = E_{3y},\quad P_{2zz} = -p_3,\quad P_{2xz} = 0$$

Thus, with account for (1.1), (1.5), (1.13) we obtain a system of linear equations in the relfection and transmission amplitude coefficients $W_{\mu\nu}$ and $W_{\mu\nu}'$ for $v_{\mu\nu}z$.

2. The solution to within the principal terms of this system for a weak magnetic field has the form

$$W_{31}' = V + \frac{1}{\rho_2\Delta\cos 2\theta_{22}}\{[(\rho_1 - \rho_2\cos 2\theta_{22}) Z_1 W_{12} + iF] N - (\rho_2\cos 2\theta_{22} - \rho_3)[M - i(N^2 - M^2)] Z_3 W_{32}' - L(1 - iM)\}$$

$$W_{11} = W + \frac{1}{\rho_2\Delta\cos 2\theta_{22}}\{(\rho_1 - \rho_2\cos 2\theta_{22})[MZ_3 - i(N^2 - M^2)Z_1] Z_1 W_{12} - [(\rho_2\cos 2\theta_{22} - \rho_3) Z_3 W_{32}' - iL] Z_1 N + F(Z_3 + iZ_1 M)\}$$

$$W_{32}' = \frac{\Phi_2}{\delta\rho_3}\{[(\rho_2\cos 2\theta_{22} - \rho_3)(1 + V)\operatorname{tg}\theta_{31} - \rho_2 P\cos(2\theta_{22} - \theta_{21})](\cos\gamma_{23}d - in\sin\gamma_{23}d) + (\rho_1 - \rho_2\cos 2\theta_{22}) W\operatorname{tg}\theta_{11} - \rho_2 R\cos(2\theta_{22} - \theta_{21})\}$$

$$W_{12} = -\frac{\Phi_2}{\delta\rho_1}\{[(\rho_1 - \rho_2\cos 2\theta_{22}) W\operatorname{tg}\theta_{11} - \rho_2 R\cos(2\theta_{22} - \theta_{21})](\cos\gamma_{23}d - im\sin\gamma_{23}d) + (\rho_2\cos 2\theta_{22} - \rho_3)(1 + V)\operatorname{tg}\theta_{31} - \rho_2 P\cos(2\theta_{22} - \theta_{21})\}$$

$$W = 2\Delta^{-2} Z_3 N,\qquad V = \Delta^{-1}\{M(Z_1 - Z_3) - i[(N^2 - M^2)Z_1 - Z_3]\},\quad \Delta = M(Z_1 + Z_3) - i[(N^2 - M^2)Z_1 + Z_3]$$

$$R = \frac{2\sin\theta_{22}}{Z_1 N}\left\{Z_3(1 + V)\operatorname{ctg}\gamma_{22}d + i\left[Z_{2\tau}\sin^2 2\theta_{22} + \frac{1 - \cos\gamma_{21}d\cos\gamma_{22}d}{\sin\gamma_{21}d\sin\gamma_{22}d} Z_2\cos 2\theta_{22}\right](1 - V)\right\} \tag{2.1}$$

$$P = \frac{2\sin\theta_{22}}{Z_1 N}\left[\frac{Z_3}{\sin\gamma_{22}d}(1 + V) - i\left(\frac{\operatorname{ctg}\gamma_{21}d}{\sin\gamma_{22}d} - \frac{\operatorname{ctg}\gamma_{22}d}{\sin\gamma_{21}d}\right) Z_2(1 - V)\cos^2 2\theta_{22}\right],\quad \Phi_2 = \frac{\psi_2\sin^2\varphi\sin\theta_{21}}{\sqrt{-i\omega\eta_2}\cos 2\theta_{22}}$$

$$F = \frac{2}{\sin\gamma_{22}d}(\rho_1 W_{12}\cos\gamma_{22}d - \rho_3 W_{32}') Z_{2\tau}\cos^2\theta_{22},\quad L = \frac{2}{\sin\gamma_{22}d}(\rho_1 W_{12} - \rho_3 W_{32}'\cos\gamma_{22}d) Z_{2\tau}\cos^2\theta_{22}$$

$$N = \frac{Z_2}{Z_1}\frac{\cos^2 2\theta_{22}}{\sin\gamma_{21}d} + \frac{Z_{2\tau}}{Z_1}\frac{\sin^2 2\theta_{22}}{\sin\gamma_{22}d},\quad M = \frac{Z_2}{Z_1}\cos^2 2\theta_{22}\operatorname{ctg}\gamma_{21}d + \frac{Z_{2\tau}}{Z_1}\sin^2 2\theta_{22}\operatorname{ctg}\gamma_{22}d$$

$$Z_\mu = \frac{\rho_\mu a_\mu}{\cos\theta_{\mu 1}},\qquad Z_{2\tau} = \frac{\rho_2 b}{\cos\theta_{22}},\quad \delta = (m + n)\cos\gamma_{23}d - i(1 + mn)\sin\gamma_{23}d$$

$$m = \frac{a_3}{a_2}\left(\frac{\psi_3\sin^2\varphi - i\omega\eta_3}{-i\omega\eta_2}\right)^{1/2},\qquad n = \frac{a_1}{a_2}\left(\frac{\psi_1\sin^2\varphi - i\omega\eta_1}{-i\omega\eta_2}\right)^{1/2}$$

Here V and W are the reflection and transmission coefficients in the absence of the magnetic field [5].

Thus, for oblique incidence of the magnetoacoustic wave on the layer the reflection and transmission coefficients differ from the conventional acoustic coefficients by terms of order $\psi_2(-i\omega\eta_2)^{-1/2}$.

Setting d = 0 in (2.1), we obtain the reflection and transmission coefficients of the interface of two media – 3 and 1 [1].

For normal incidence the magnetic field has no effect on the amplitude coefficients in the approximation considered.

3. The solution of (1.13) for a strong magnetic field has the following form to within terms of order $1/\sqrt{\psi}$

$$W_{31}' = \frac{1}{\Delta}[n_2(n_1 - n_3)\cos\gamma_{21}d - i(n_2^2 - n_1 n_3)\sin\gamma_{21}d]\quad W_{11} = \frac{2}{\Delta}n_2 n_3 \tag{3.1}$$

$$W_{12} = \frac{2n_2 n_3\operatorname{tg}\varphi}{a_1\sqrt{\psi_1}\Delta\delta}\{[(q + \xi L)(m_2^*\cos\gamma_{22}d - im_3\sin\gamma_{22}d)$$

$$-m_2^*(r+\xi L)\cos\gamma_{21}d]\cos(\theta_{11}-\varphi)+i\frac{rm_2^*}{\cos\theta_{21}}\left[\sqrt{\frac{\rho_2}{\rho_1}}\cos\varphi+\sin\theta_{21}\cos(\theta_{11}+\varphi)\;+\xi\frac{n_1}{n_2 r}L\cos\theta_{21}\cos(\theta_{11}-\varphi)\right]\sin\gamma_{21}d\Big\}$$

$$W_{32}'=-\frac{1}{m_2^*}\left\{W_{12}(m_2^*\cos\gamma_{22}d-im_1\sin\gamma_{22}d)+iW_{11}\frac{\operatorname{tg}\varphi}{a_1\sqrt{\psi_1}}(q+\xi L)\cos(\theta_{11}-\varphi)\sin\gamma_{22}d\right\}$$

$$\Delta=n_2(n_1+n_3)\cos\gamma_{21}d+i(n_2^2+n_1n_3)\sin\gamma_{21}d,\quad \delta=m_2^*(m_1+m_3)\cos\gamma_{22}d-i(m_2^{*2}+m_1m_3)\sin\gamma_{22}d$$

$$q=m_2^*a_2-m_1a_1,\qquad r=m_2^*a_2-m_3a_3,\quad n_\mu=\sqrt{\rho_\mu}\cos\theta_{\mu 1},\qquad m_\mu=\rho_\mu a_\mu,\qquad m^*=m_2/\sqrt{1+\xi\operatorname{ctg}^2\varphi}$$

$$L=\frac{2m_2a_2}{\cos(\theta_{11}-\varphi)}\sin\theta_{11}\sin\varphi,\quad \gamma_{21}=\frac{\omega\cos\theta_{21}}{a_2\sqrt{\psi_2}},\qquad \gamma_{22}=\frac{\omega}{a_2\sqrt{\sin^2\varphi+\xi\cos^2\varphi}}$$

If the layer thickness is considerably less than the length of the fast magnetoacoustic wave ($\gamma_{21}d \ll 1$), (3.1) simplify considerably

$$W_{31}'=\frac{n_1-n_3}{n_1+n_3}-2i\frac{n_2^2-n_1^2}{(n_1+n_3)^2}\gamma_{31}d,\qquad W_{11}=\frac{2n_3}{n_1+n_3}+2i\frac{n_2^2+n_1n_3}{(n_1+n_3)^2}\gamma_{31}d$$

$$W_{32}'=\frac{\beta}{\sqrt{\psi_1}\delta}[(r+\xi L)(m_2^*\cos\gamma_{22}d-im_1\sin\gamma_{22}d)-(q+\xi L)m_2^*]$$

$$W_{12}=\frac{\beta}{\sqrt{\psi_1}\delta}[(q+\xi L)(m_2^*\cos\gamma_{22}d-im_3\sin\gamma_{22}d)-(r+\xi L)m_2^*]$$

$$\beta=\frac{2n_3\operatorname{tg}\varphi}{a_1(n_1+n_3)}\cos(\theta_{11}-\varphi),\qquad \gamma_{31}=\frac{\omega\cos\theta_{31}}{a_3\sqrt{\psi_3}}$$

Setting $\xi = 0$ in (3.1) and (3.2), we obtain the reflection and transmission coefficients for a liquid layer [1].

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